

Teacher Action Research: Teaching Self-regulation in the Context of Learning Multi-digit  
Multiplication in Upper Elementary Special Education Students

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by

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### **Statement of the Problem**

In 2017, 60 % of 4th grade students and 67 % of 8th grade students performed below “proficient” on the National Mathematics Achievement Test (National Assessment of Educational Progress, 2017). Furthermore in 2015, 4th grade students in the United States ranked 35 in industrialized nations in mathematics achievement; while 8th graders ranked 25th (U.S. Department of Education, National Center for Education Statistics, 2017). Considering these statistics, teachers may need to consider different methods for teaching mathematics.

Research shows that learning multi-digit multiplication and division computation strategies based on operation properties and knowledge of place values that can be understood or flexibly derived by students is an effective strategy to teaching these complex skills in special education (Baroody & Dowker, 2003; Woodward, 2006; Schulz, A. 2018). In fact, research demonstrates that advanced calculation strategies based on conceptual reasoning (which enables metacognitive regulation) can lead to automaticity of mathematical computations (Baroody & Rosu, 2004; Woodward, 2006; Crowley, Shrager, & Siegler, 1997). Even though applying digit-based procedural memorization to solve multi-digit computation is often difficult for higher grade students (Anghileri, 2001; Hickendorff, 2013), many teachers still rely on this traditional approach. (Ambrose, Baek, & Carpenter, 2003).

Another consequence of the aforementioned traditional approach is that it does not foster self-regulated learning skills that are often lacking in struggling students (Butler & Schnellert, 2015). Self-regulated learning is typically described as the ability of a student to independently manage his or her own behavior, cognition and environment in order to set, pursue, monitor, and adjust the use of a strategy to achieve an academic goal (Reeve, Ryan, Deci, & Jang, 2008; Buzza & Dol, 2015; Buzza & Allinotte, 2013). Teaching students how to self-regulate can lead

to success in and out of school (Cleary, 2015; Winne & Hadwin, 2008; Zimmerman & Schunk, 2001; Boekaerts, Pintrich & Zeidner, 2000; Schunk & Zimmerman, 2008).

Teachers usually do not teach self-regulation skills because they feel they do not have the time (Wehmeyer, Agran, & Hughes, 2000). This is unfortunate because developing these skills can help increase self-efficacy and motivate struggling students to remain academically engaged (Solberg et al., 2012; Maag, et a. 1993 and Maag 1992; Schunk & Ertmer, 200). Furthermore, research shows positive short-term and long-term outcomes when students are taught strategies to develop self-regulation. The short-term outcome of self-regulation interventions is increased academic accuracy, productivity, and on-task behavior for students with ADHD (e.g., Reid, Trout, & Schartz, 2005), learning disabilities (e.g., Reid, 1996), and emotional-behavioral disorders (Mooney, Ryan, Uhing, Reid, & Epstein, 2005). The long-term outcome of teaching self-regulation skills is that it can reinforce the underlying, mutually supporting attributes that are needed for self-regulation described in Table A1 (Montague & Applegate, 1993a; Montague & Applegate, 1993b; Swanson & Jerman, 2006; Montague, 2007).

## **Review of the Literature**

### **Research on Self-Regulation in Mathematics**

In order to develop effective self-regulated learning interventions for multi-digit multiplication instruction, I analyzed research on effective self-regulation interventions with an emphasis on math instruction for struggling students. Many of the studies on this topic are single case studies with multiple baselines. Although the generalizability of single-case research can be limited, when the cumulative body of research is considered, there are clear guidelines for effective self-regulation intervention strategies (Montague, 2007). First, I describe what researchers have determined to be the most effective components to self-regulation intervention followed by more detailed information on how to help students set goals and apply metacognitive processes.

Different self-regulatory strategy instruction techniques have been developed by researchers for elementary, middle, and high school level. Most of them follow the same basic steps and vary only in the content of the checklists and the difficulty of the material covered (Montague, 2007). According to Montague (2007), the following are the basic components to effective self-regulation interventions for math:

- Collaborate with student to determine goals and establish a baseline performance level.
- Model self-regulation strategies in context.
- Have students verbalize self-regulation strategies.
- Provide self-recording cards, cue cards, or prompt sheets that students can use to self-monitor or self-instruct until they are successful at accurately completing the task.
- Have students maintain a visual record of progress.
- Fade cues and prompts as students become more competent in using self-regulation.

One component of self-regulation is self-instruction. Self-instruction is when a student verbalizes the steps they need to perform (using a checklist as a guide, if needed).

Self-instruction has been shown to improve accuracy, productivity and generalization for solving math computation problems for elementary students with learning disabilities (Wood, Rosenberg & Carran, 1993) and is associated with the development of metacognitive skills (Kroesbergen and Van Luit, 2003). Furthermore, research has suggested that teacher mediated self-coping peer modeling in which students watch their peers effectively self-monitor can be one of the most useful instructional methods for struggling math students since it can increase both self-efficacy and accuracy (Schunk & Hansen, 1989, 2013; Schunk Hanson, & Cox, 1987; Zheng, Flynn & Swanson, 2013).

### **The Role of the Teacher When Developing Motivation and Metacognition**

Although it is common for teachers to view themselves as purveyors of information who are primarily in control of their students' academic activities, research has shown that when teachers shift their role more towards a collaborator and cognitive coach, students are more likely to develop self-regulation skills (Buzza, & Dol, 2015). For example, several studies have indicated that middle school and high school students with LD benefit from strategy instruction that promotes metacognitive processes, specifically in mathematics (e.g., Hutchinson, 1993; Maccini & Hughes, 2000; Montague, 1997a, 1997b).

Butler, Beckingham and Lauscher's (2005) research demonstrated how teachers can guide students to utilize metacognitive skills to develop their own strategies for solving problems. This is primarily done by using questioning that encourages the student to articulate their understanding and develop their own strategies that they can then record for future use. The teacher questioning and the students explaining also elucidates their misunderstanding and

refines their metacognitive skills. Table A2 provides examples of the types of questions that teachers can encourage students to start asking themselves at each stage of self-regulation (adapted from Butler, et. al. 2005).

### **Effective Goal Setting**

Effective goal setting informs all other aspects of self-regulation and can enhance self-efficacy, motivation, focus and achievement. (Chadsey-Rusch, 1992; Puustinen & Pulkkinen, 2001; Schunk & Ertmer, 2000). The most effective goals are specific, challenging, short-term, and valued by the student (Bandura, 1989; Locke & Latham, 1990; Locke, Shaw, Saari, & Latham, 1981). Goals should also ensure students can experience success so that students can develop a sense of self efficacy which usually results in improved motivation and focus (Schunk & Ertmer, 2000).

Furthermore, research suggests goals set with a mastery-orientation (i.e., focus on personal improvement and mastery of concepts) rather than performance orientation (e.g., focus on grades or competition) leads to positive self-regulatory behaviors in students with learning disabilities (Sideridis, 2005). Customized checklists, and cue cards are an effective way to clearly define measurable goals and enhance strategy implementation, self-instruction, self-monitoring and self-evaluation (Dunlap and Dunlap, 1989).

### **Purpose and Research Question**

As a special education teacher at a small private school, many of my past students and all current upper elementary students have significant difficulty in self-regulation and are lacking in most, if not all, of the underlying attributes that support self-regulated learning. I have been primarily trained to use direct, explicit instruction. Given the emphasis on direct skill instruction, there is less time to devote instruction to self-regulation skills that will contribute to their

confidence, motivation and success in many areas of their lives (Cleary, 2015; Winne & Hadwin, 2008; Zimmerman & Schunk, 2001; Boekaerts, Pintrich & Zeidner, 2000; Schunk & Zimmerman, 2008).

The purpose of this action research is to use interventions that provide an opportunity for my students to apply self-regulation skills in the context of learning multi-digit multiplication. Therefore, my research question is: What is the effect of instruction designed to enhance self-regulation on students' academic achievement in solving multidigit multiplication?

### **Specific Guidance on Teaching Multi-digit Multiplication Strategies**

Following the Concrete Representational Abstract (CRA) sequence is an effective approach for teaching struggling students how to solve calculation problems (Flores, Hinton, & Strozier, 2014). It involves the following steps: (1) start with demonstrations using concrete materials (e.g., place value blocks), (2) provide explicit instruction to teach students how to use representational images to convey the concept (e.g., drawings), and (3) teach students how numbers and symbols are used to convey the concept in the abstract (Flores, et al., 2014).

Bobis (2007) described using the area model approach, which involves applying the commutative property of addition and multiplication and the distributive property to help a struggling 6th grade school student learn how to solve multi-digit multiplication problems. An example is provided in Appendix A. The area model approach would be particularly helpful for my students not only because it can make sense and thus elicit metacognition, it also decreases the need for memorizing abstract procedures and enable students with low math fact fluency to use the tools they need for computation.

## **Method**

### **Setting and Student Background**

I teach three students their core classes, including mathematics, at a small private school. Pseudonyms have been used to protect their privacy. Anne and Michelle attend full time, while Darlene is attending school Mondays, Tuesdays, Wednesday and Fridays while Thursdays she is homeschooled. Table A3 describes each student based on formal assessments and other student records.

These students have been working on applying place value and the base ten system to solve addition and subtraction problems, but they have not mastered solving complex applications of these computations. For example, they have difficulty mentally adding numbers like  $230 + 100$  without prompts to remind them that they can do it without paper and pencil. They are familiar with the commutative property of addition and multiplication and how to show single digit multiplication with arrays, but at this time, have not learned about the distributive property.

Many of my past students and all of current upper elementary students are lacking in some of the underlying attributes that support developing self-regulated learning described in Table A1. This affects their ability to successfully implement the three steps of regulation described by Butler et al. (2005). Table A4, describes my perception of their current level of performance on these three steps based on student records and observation.

Since they all have difficulty with math accuracy and/or fluency, executing multistep processes and describing their mathematical thinking, learning multi-digit multiplication will provide them the necessity and opportunity to apply a variety of effective self-regulation skills to



perform the three main steps to self-regulation by asking themselves effective questions, as described by Butler et al (2005).

### **Teaching Methodology**

I used the area model described above since it requires less math fluency and procedural memory; it eases the use of tools and scaffolds. In addition, since it is conceptually-based it eases the use of the CRA steps, recommended by Flores, et al. (2014). These conditions provided students the opportunity to apply their metacognitive skills to effectively plan, perform and self-evaluate. First, I completed the following steps to pre-teach or review of foundational concepts:

- Provided daily review of multiplication facts using the adaptive program on Math Facts Pro.com for a few months prior to lesson.
- Reviewed adding numbers based on 10s (e.g.,  $120 + 200 = ?$ ;  $1000 + 550 + 8 = ?$ ).
- Reviewed the following vocabulary: commutative property of addition and multiplication, addend, factors, products, sum. These are stored in their math reference notebook and on a word wall.
- Reviewed numbers in their expanded form (e.g.,  $37 = 30 + 7$ )
- Pre-taught multiplying single digit numbers by numbers based on ten (e.g.,  $10 \times 5$ ;  $2 \times 300$ ); the distributive property using visual supports and manipulatives; the mathematical meaning of area and how to calculate it.

After conducting a pre-interview that includes a probe (Figure B1), I reviewed the “Success for School and Beyond” poster that outlines the types of questions that students ask themselves in each step of regulation (Figure C1). I explained that they will be practicing asking themselves these questions.

Next, I provided direct instruction followed by guided practice applying the area model for solving multidigit multiplication problems using the CRA model proposed by Flores, et al. (2014) by using graphic representations, then graphic organizers (examples provided in Figures C2, C3, C4).

After the students understood the area model concepts, I provided direct instruction via think-aloud demonstrations and guided practice using all of the scaffolds including reference sheets (Figure C2), graphic organizers (Figures C3 and C4), the “Silly Mistake” checklist (Figure C10) and multiplication chart to answer some questions. Likewise, I modeled how to explain work using mock student examples with mistakes (Figures C7 and C8) using the word wall (Figure C9). Finally using these mock student examples, I provided direct instruction and guided practice on filling out the self-reflection and goal setting sheets (Figures C5, C6). When introducing the self-evaluation and goal setting sheets, I referred to the “Success for School and Beyond” poster (Figure B1) and then provide the following explanation:

Not only will you be learning a different way to multiply multiple digits that will hopefully make more sense to you, you will be learning how to ‘work smarter, not harder’ by learning how to approach mistakes and challenges in such a way that you can learn from them. You will also be learning how to show and explain your work and help each other identify what tools you need to be successful. Because of this you will be doing fewer problems, but then you will talk about how you solved them and evaluate how you might improve or challenge yourself when you’re ready using these self-evaluation and goal setting sheets. The self-evaluation and goal setting sheets will also be graded since it is so important to reflect on your challenges and successes whenever you are learning new things.

## **Student modeling, Self-instruction and Self-Monitoring**

Since research indicated that students benefit from teacher-mediated peer modeling, (Schunk & Hansen, 1989, 2013; Schunk Hanson, & Cox, 1987; Zheng, Flynn & Swanson, 2013) self-instruction (Schunk & Hansen, 1989, 2013; Schunk Hanson, & Cox, 1987; Zheng, Flynn & Swanson, 2013) and self-monitoring (Butler, et. al., 2005). I employed the following steps after the completion of the four problems on a daily basis:

1. First I described how to solve one problem on the board, then the students took turns describing how to solve the remainder of the problems. In the case they had different problems because they progressed to  $2 \times 3$  multiplication at different rates, each student still presented but I checked for remaining problems for accuracy without telling them where the errors were so that they had an opportunity to find their own errors.
2. I provided prompts, if needed to get them to explain their reasoning using mathematical language. Examples of prompts include: What property allowed you to write the numbers in the expanded form on the area model? What do you call the numbers inside the area model again? What operation did you use to fill in the grid? What operation did you use to fill find the final product?
3. Students who solved a problem incorrectly made the corrections on their paper using a pen and a highlighter.
4. They then filled out the self-reflection and goal sheet (Figures C5, and C6).
5. After they independently filled out the self-reflection sheet, I reviewed it with them and put a “P” for prompt, if it is necessary to give a specific prompt.
6. They filled out the Goal Setting Sheet on Fridays only, leaving the bottom portion for Tuesday-Thursday blank.

An example of a graded self-reflection and goal setting sheet are provided in Figures C.

The daily self-reflection and goal setting sheets are designed to provide a scaffold for students for the type of questions that students who self-regulate ask themselves, as described by Butler, et. al. (2005) and presented in Table A2. Furthermore, Dunlap and Dunlap (1989) suggested that checklists are an effective way to clearly define measurable goals and enhance strategy implementation, self-instruction, self-monitoring and self-evaluation; all traits of a student with strong metacognitive skills.

## **Data Collection**

### **Semi-structured Interviews and Baseline Probe**

I conducted a semi-structured student pre- and post- interviews as described in Appendix B. The pre-interview was to determine their level of knowledge and exposure to multidigit multiplication. The post interview was intended to determine their attitudes and opinions regarding learning multidigit multiplication using the area model.

### **Daily Computation Accuracy**

Daily computation accuracy on multi-digit multiplication problems were measured based on a total of 10 points per problem, 4 problems per day, 4 times per week over 3 weeks. The grand total of possible points will be 40 for 4 problems per day based on the following parameters:

1. Factors correctly placed on grid in expanded form: 2 points for 2 X 2; not graded for 2 X 3.
2. Correct partial products filled in grid: 2 points for each correct partial product

3. Addition of partial products: 2 points for lining up correctly, as applicable; 2 points for correct final answer. Total points: 4. If student uses mental math and correctly solves the problem, they received 4 points.

An example of grading is provided in Figures C3 and C4.

### **Comparison of percent accuracy to first semester tests.**

In order to compare daily computation accuracy of the daily multidigit multiplication problem sets (4 problems per day/ 4 times per week for 3 weeks) with previous academic achievement, I computed the percent accuracy of the entire first semester for the same three students on 12 weekly review tests. The weekly review tests contained 8-10 problems that were directly based on the previously learned concepts and identical in nature to the previous 5 daily problems sets that went home for homework. These problem sets were checked daily, marked for student correction, and reviewed when necessary. There were no more than two new review concepts put on the homework each week.

### **Daily Task Completion**

Daily task completion was measured as the percent completed on 4 problems.

### **Self-Reflection Accuracy**

After correcting their work, students were required to fill out the self-reflection that included error analysis, as shown in Appendix A and I reviewed it with them and kept track of the number of prompts they needed to accurately fill it out. Self-reflection accuracy was completed daily and measured as a percent. The percent was determined giving 2 points to each of the 10 questions/sub-questions and subtracting 1 point for each prompt given to the student.

### **Teacher Observation**

After each week, I wrote a brief description of what I did and how the students responded in terms of demonstrating more self-regulation attributes, their attitudes and behavior.

After three weeks of implementing this plan, I conducted a post-semi structured interview with students as described in Figure B2.

### **Data Interpretation Methods**

In order to determine the effect of instruction designed to enhance self-regulation has on students' academic achievement in solving multidigit multiplication, the following data was analyzed:

- Percent accuracy graphed with a trend line.
- Percent task completion
- Summary of student observations during student presentations.
- Self-Reflection Sheet Grades
- Pre- and Post- Semi-structured Interviews.

In addition, a comparison was made of the percent accuracy of 12 daily practice of multidigit multiplication with progressively more difficult problems to 12 weekly review tests from Fall 2018 semester.

## **Results**

### **Semi-structure Pre-Interview and Probe - Key Findings:**

All three students could solve  $21 \times 6$  using the traditional method, but they could not solve  $49 \times 6$  due to carry-over addition errors. None of the students demonstrated an ability to multiply  $37 \times 58$ . Darlene expressed that she was scared to learn how to multiply multi-digit problems again. When solving the problems, she chose to use her flash cards to find math facts rather than the multiplication table that was on the desk. Even though Michelle had learned the area model method for multidigit multiplication, she attempted to solve all the problems using the traditional method. The graphic organizer she used to solve these problems was on the desk in front of her and I asked her if there were any tools that she might need to solve the problems and she responded “No”.

### **Daily Accuracy, Task Completion and Self-Reflection Accuracy Graphs**

Daily Accuracy, Task Completion and Self-Reflection Accuracy are reported as a bar graph with a trend line for each student to provide a visual representation of academic growth and the relationship between these factors (Appendix D). Changes in the difficulty of the problems are noted below each graph. In addition, I compared their average math performance the previous semester on weekly math tests that are directly tied to mastered “review” material and daily homework from the prior week.

All students showed a positive trend in academic achievement and self-reflection accuracy; however, I noticed that when they got problems incorrect, all three needed prompts to find their errors, which is an important self-regulation skill they will need to continue to work on. All students consistently scored 100% on task completion, which was expected by Darlene and

Annie, but not by Michelle, who typically doesn't complete work at least twice in a two-week period.

When comparing average percent accuracy on these progressively more difficult 12 daily problem sets based on a new skill with previous academic achievement on 12 weekly review tests from last semester, all of them earned a higher percentage on average on the former as shown in Appendix E. Darlene's average was 17 points higher; Annie 10 higher and Michelle 8 higher. The weekly review tests were directly based on previous 5 days of homework (which is reviewed daily and is based on previously taught concepts). There were no more than two new review concepts put on the homework each week.

### **Teacher Observation**

**Use of tools.** Annie consistently used the multiplication chart as a tool for more difficult problems. Darlene would finish her work first, didn't use her multiplication chart and had several "silly errors" the first week. I pointed this out to her. The following weeks she slowed down and used tools like the silly mistake checklist and the multiplication chart, but not as much as I think she should have. She continued to consistently finish the assignments before the other students, one of whom was also working diligently.

**Student Presentations.** On the first day, it took 2 hours to answer the questions, review the problems, and fill in the self-reflection and goal sheets largely due to the number of prompts that were needed during their presentation. By the beginning of Week 2, it only took 1 hour to complete all these steps. Initially, Darlene and Michelle were excited to share; while Annie was nervous and quiet.

Michelle made some unintentional critical comments about Darlene on day 1 and this caused Darlene's confidence to present declined markedly on the following 2 days. I spoke



privately with Michelle to explain why she should keep critical thoughts to herself. The next day, Michelle told the class that they were doing really well and how hard it was for her when she first started doing multidigit multiplication. She sang a song called “You’re only human.” Michelle started to purposefully make mistakes so that the other students could “catch” her and get “points”. All students made this a custom and consistently created common mistakes for their classmates to fix. I also started assigning Darlene presentation problems that she got correct so that she could explain her work without fear of being caught having made an unintentional mistake. By the beginning of week 3, Darlene was able to confidently describe all the steps without prompting. The other two students continued to need prompts but were able to respond without looking at the word wall by week 3.

Initially students had questions pertaining to the placement of the partial factors on the area model grid and the order for adding partial products. It was a good opportunity to review the commutative property of addition and multiplication. This was confirmation of one of the benefits of students demonstrating their work: clarifying and deepening their understanding. Darlene and Annie also had difficulty putting the partial products in the correct boxes (like the multiplication tables).

It is also interesting to note that I had to encourage my students to challenge themselves once they demonstrated mastery.

**Semi-structured Post-Interview.** All three students preferred learning multidigit multiplication using the area model rather than the traditional method because it’s “way easier” and it “makes me feel smarter” (Darlene and Annie, respectively). They all felt they understood how to do the problems “really well”. In terms of what they felt was most difficult for them, Darlene and Annie both thought multiplying the factors to get the partial products was the

hardest. Darlene realized that slowing down and looking for silly mistakes was helpful. Annie said that using her fingers to multiply helped and checking her work before going to the board helped her. Michelle said what helped her the most was thinking “If you don’t get this done, you won’t get your electronics or IPAD.” Interestingly, none identified other reference sheets, explaining your work or correcting your own errors as helpful.

## **Discussion**

The results of this teacher action research suggest that employing research-based multiplication instruction that facilitates the development of self-regulatory skills can improve academic achievement and accuracy in upper elementary students who struggle in math and self-regulation. Specifically, struggling math students can benefit from learning multidigit multiplication using the area model based on the concrete, representational, abstract model recommended by Flores, et al. 2014 combined with being involved in setting mastery-based goals (Sideridis, 2005), self-instruction and peer modeling (Wood, Rosenberg & Carran, 1993; Kroesbergen and Van Luit, 2003), self-monitoring progress (Schunk & Hansen, 1989, 2013; Schunk Hanson, & Cox, 1987; Zheng, Flynn & Swanson, 2013) and utilization of scaffolds (Montague, 2007; (Dunlap and Dunlap, 1989).

## **Limitations**

Being the teacher and researcher and including some qualitative data, this research has a large potential for bias. Furthermore, the small population size (3) and sample size (12) do not allow for rigorous statistical analysis or generalization.

## **Future Research**

Considering the facts that students are behind a large number of industrialized nations in mathematics achievement (U.S. Department of Education, National Center for Education Statistics, 2017) and the importance of supporting self-regulation in a wide variety of struggling students (Butler & Schnellert, 2015; Reid, Trout, & Schartz, 2005; Reid, 1996, Mooney, Ryan, Uhing, Reid, & Epstein, 2005; Montague & Applegate, 1993a; Montague & Applegate, 1993b; Swanson & Jerman, 2006; Montague, 2007), it is worthwhile to conduct more research on mathematics instruction that enhances self-regulation. It would be particularly beneficial to

conduct research on this topic on a larger scale since most of the research on enhancing self-regulation during math instruction is based on case studies and therefore are not generalizable (Montague, 2007).

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## Appendix A

Table A1

*Underlying Attributes and Skills that are Necessary for Successful Self-Regulation*

Attribute	References
Self-efficacy: the belief in the one's ability to be successful at something. Increased self-efficacy increases motivation.	Schunk and Zimmerman 2008; Boekaerts, Koning & Vedder, 2006
Accurate achievement attributions.	Schunk & Zimmerman, 2008
Executive function: the ability to focus, plan, organize and implement one's efforts to achieve a goal	Bryan, Burstein, & Bryan, 2001
Metacognition: the ability to reflect on one's mental process and adjust strategies accordingly	Boekaerts, 1997, Winne & Perry, 2000; Belfiore & Hornyak, 1998

Table A2

*Steps of Self-Regulation and Corresponding Examples of Self-Questioning*

Steps	Questions Students May Ask Themselves
Plan	What is my goal? Is it easy or hard for me? What tools and resources do I need to accomplish this goal? What strategy am I going to use? Do I need to ask for help? What parts of this problem are tricky or confusing, which parts make sense?
Perform and Self-Monitor	Am I completing the task? Am I effectively applying a strategy? Do I need to modify or use a different strategy?
Self-Evaluation	Did I accomplish my goal? Why or why not? What strategies or tools worked or didn't work? Why? Should I modify my strategy?

Note: Questions adapted from by Butler et al, 2005.

Table A3

*Student Background*

Name	Age (years)	Weekly attendance	Disability or Challenges	Current Math Performance
Anne	10	4 days	Autism, Speech Delay. Very low Math and Oral Expression Fluency. Low working memory. Slow processing. Resists verbal self-instruction for spelling practice. Becomes very inhibited verbally when recorded. ELL.	3rd grade level. Not fluent but accurate in multiplication, addition and subtraction but knows most facts. Learned traditional 2-digit by 1-digit multiplication but doesn't remember how to do it. Currently becoming familiar with division facts.
Darlene	12	2 days	Nonverbal Learning Disability. Speech and Language Delay. Very low working memory. Resists verbal self-instruction for spelling practice. ELL.	2 grade level. Not accurate or fluent in addition or subtraction. Knows, 78 out of 121 multiplication facts for 0-10. Has been introduced to traditional 2 digit by 1-digit multiplication but is unable to correctly solve these problems consistently.
Michelle	11	4 days	Aspergers/ADHD Excellent verbal skills Takes medication for anxiety and ADHD. Math identified as causing "extreme" anxiety. Visual and auditory processing challenges. Difficulty following directions. Difficulty asking for and taking breaks when needed. Difficulty with whole body listening due to trichotillomania caused by medication.	3rd grade level Highly accurate with addition and subtraction facts but not fluent. Knows most multiplication facts for factors 0-11 but is not fluent. Knows division facts for the following divisors: 1, 2,3, 4, 5, , 10, 11. Has learned both methods of multidigit multiplication and prefers the area model. Complex math concepts are pre-taught 1-1 to reduce classroom anxiety.

Table A4

*Present Level of Underlying Attributes and Skills that are Necessary for Successful Self-Regulation*

Attribute	Current Level of Performance: High/Medium/Low		
	Anne	Darlene	Michelle
Plan	Low Has difficulty identifying learning goal.	Low Has difficulty identifying learning goal and the level of difficulty of problems.	Low Becomes anxious when a new learning goal is presented. Math identified as an “extreme” source of anxiety in student file.
Perform and Monitor	Medium Good task completion but poor monitoring. Requires oversight to accurately follow through on multi-step processes. Frequently doesn’t use tools. Strong resistance to oral explanation of mathematical thinking.	Low Good task completion but poor monitoring. Requires oversight to accurately follow through on multistep processes. Doesn’t realize when she needs to ask for help or use a tool/strategy. Frequently makes careless mistakes. Frequently doesn’t use tools.	Low Frequently requires oversight and timed 2-minute breaks to stay focused on lessons and assignments due to self-stimming behavior. Frequently doesn’t ask for help or use a tool/strategy. Motivation fluctuates from low to high on a hourly basis based on whether she is distracted by something else or if she perceives a task will be too difficult or if she is not happy with how she is doing. Self-monitors for accuracy.
Self Evaluation	Low Frequently erases answers before figuring out where the mistake occurred.	Low Frequently erases answers before figuring out where the mistake occurred.	Medium Rarely appropriately asks for breaks or help when needed. Frequently erases answers before figuring out where the mistake occurred.

## Appendix B

### Semi-structured Pre- and Post- Interview

Have you ever learned multi-digit multiplication like these? (Show them the following problems:  $21 \times 6$ ;  $49 \times 6$ ;  $37 \times 58$ )?

- a. If yes, ask the following questions:
  1. How well did you understand how to do it? Really well, Sort of, Not at all?
  2. What tools did you use? (Provide them with any tools they state.)
  3. Do you think you still remember how to do it?
  4. Are you scared or excited to learn it again?
- b. Probe: Ask students to solve the following problems:  $21 \times 6$ ;  $49 \times 6$ ;  $37 \times 58$ . If they are able to solve any of them, ask them if they understand how they got the answer. If they get  $37 \times 58$  correct, ask them to solve the following four problems:
  1.  $45 \times 76$
  2.  $32 \times 66$
  3.  $82 \times 69$
  4.  $714 \times 96$
  1. Evaluate students' answer in terms of the following components:
    1. Traditional or Area Model
    2. Ability to explain why they do each step when prompted.
      - a. Able to explain it with the following mathematical terms (e.g. (with/without prompts): List terms used and whether they were given prompts or not. Examples:
        - i. Partial products
        - ii. Expanded form
        - iii. Distributive property
        - iv. Factors
        - v. Final Products
        - vi. Place value
        - vii. Digit
        - viii. Non-zero number
      - b. Able to explain how to do it without using mathematical terms.
      - c. Not able to explain it.
    3. Level of comfort performing the steps. Easy/Medium/Hard
    4. Note any scaffolds used (e.g. multiplication table or graphic organizer).

*Figure B1.* Teacher-student semi-structured probe and pre-interview.

Appendix B (*Continued*)

Semi-structured Pre- and Post- Interview

1. How do you feel about learning multi-digit multiplication using the area model? (Prompt: Did you like it? Hate it? Did you like the traditional method better? If yes, why?)
2. How well do you think you understand how to do multi-digit multiplication? Really well, Sort of, Not at all?
3. What do you think was the hardest for you or what do you think got in your way? (Note if following prompts are needed: Did you ever feel overwhelmed or distracted and gave up? Did you keep making silly mistakes?)
4. What helped you? Use prompts below if needed (Note Prompts)
  - a. Graphic organizers?
  - b. Reference sheets (multiplication tables, check lists, etc)?
  - c. Explaining your work?
  - d. Finding and correcting your own errors?
5. We are going to learn long division using a similar method that to the area method of multiplication instead of the traditional method. How do you feel about that?

Note: Students will be asked these questions after we have completed 3 weeks of independent practice on multidigit multiplication to determine if there is any growth in their ability to implement the three steps in self-regulation: 1) plan 2) perform and monitor 3) self-evaluate. In addition, it will provide information on their attitudes towards this approach to teaching.

*Figure B2.* Post-interview questions.

Appendix C

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

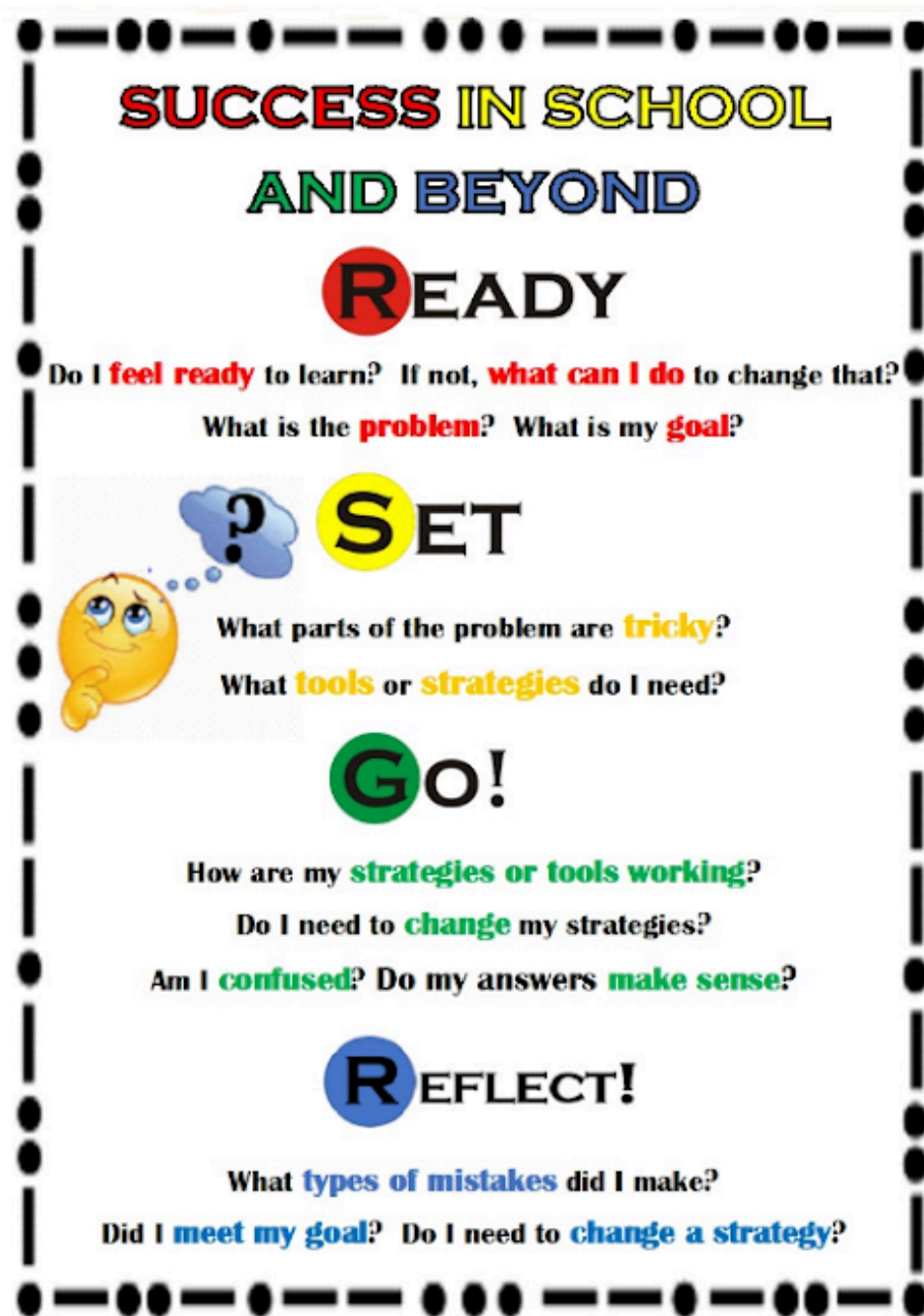


Figure  
“Success in School and Beyond” poster to explain self regulation.

CI.

Appendix C (Continued)

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

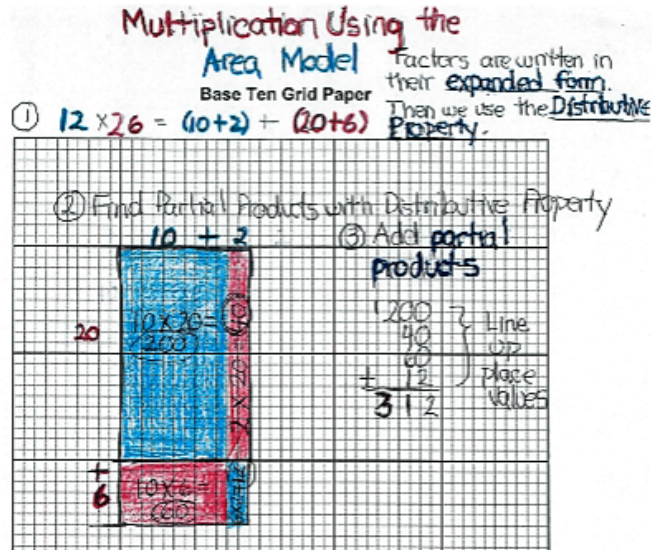


Figure C2. Guided practice example for student-created reference sheet.

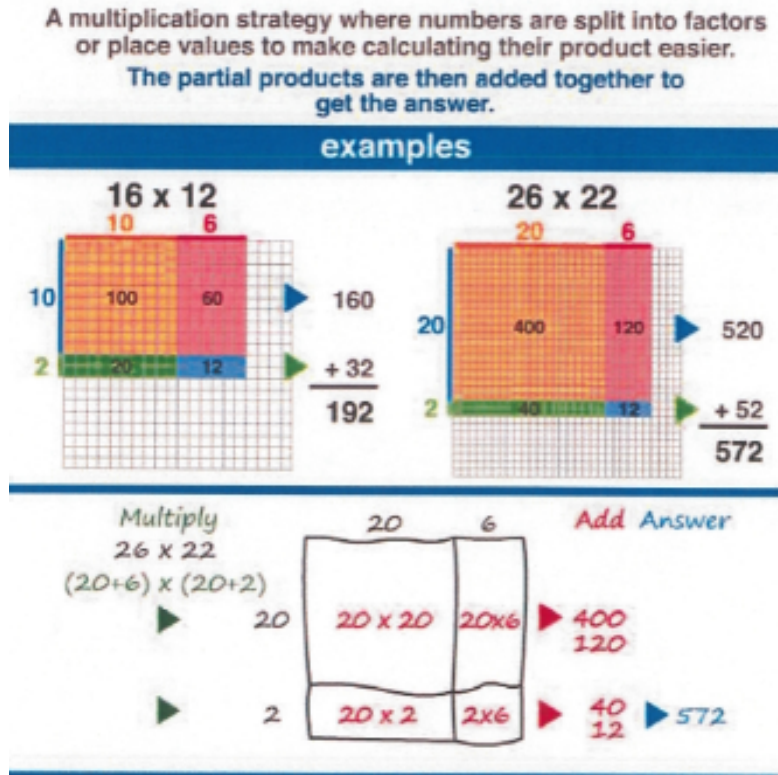


Figure C3. Graphic representations of the area model for direct instruction and guided practice.

Appendix C (Continued)

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

1-19-19

<p>① <math>34 \times 57 = ?</math></p> <p>Expanded Form  <math>(30 + 4) \times (50 + 7)</math>  <math>\times \underline{50 + 7}</math></p> <table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">30</td> <td style="padding: 2px 5px;">1500</td> <td style="padding: 2px 5px;">+ 210</td> <td style="padding: 2px 5px;">✓</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">200</td> <td style="padding: 2px 5px;">28</td> <td style="padding: 2px 5px;">✓</td> </tr> </table> <p style="text-align: right; color: red; font-size: 0.8em;">+5</p> <p style="margin-left: 20px;">+          + 1,700 +2          + 210 lining up          + 28</p> <p style="margin-left: 20px;">} partial products</p> <p style="margin-left: 20px;">1,938 -2 Final product</p> <p style="text-align: center; border: 1px solid black; width: 40px; margin: 0 auto;">7</p>	30	1500	+ 210	✓	4	200	28	✓	<p>② <math>65 \times 44 = ?</math></p> <p><math>(60 + 5) \times (40 + 4)</math>  <math>\times \underline{40 + 4}</math></p> <table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">60</td> <td style="padding: 2px 5px;">2400</td> <td style="padding: 2px 5px;">240</td> <td style="padding: 2px 5px;">✓</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">200</td> <td style="padding: 2px 5px;">20</td> <td style="padding: 2px 5px;">✓</td> </tr> </table> <p style="text-align: right; color: red; font-size: 0.8em;">+6</p> <p style="margin-left: 20px;">2,420 +2 lining up          + 200 +240          2,620 -2 incorrect answer</p> <p style="text-align: center; border: 1px solid black; width: 40px; margin: 0 auto;">6</p>	60	2400	240	✓	5	200	20	✓
30	1500	+ 210	✓														
4	200	28	✓														
60	2400	240	✓														
5	200	20	✓														
<p>③ <math>29 \times 81 = ?</math></p> <p><math>(20 + 9) \times (80 + 1)</math></p> <table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">20</td> <td style="padding: 2px 5px;">1600</td> <td style="padding: 2px 5px;">20</td> <td style="padding: 2px 5px;">✓</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">9</td> <td style="padding: 2px 5px;">720</td> <td style="padding: 2px 5px;">9</td> <td style="padding: 2px 5px;">✓</td> </tr> </table> <p style="text-align: right; color: red; font-size: 0.8em;">+3</p> <p style="margin-left: 20px;">169 +2 (line up)          + 92          171 -2 wrong answer</p> <p style="text-align: center; border: 1px solid black; width: 40px; margin: 0 auto;">5</p>	20	1600	20	✓	9	720	9	✓	<p>④ <math>\quad \times \quad = ?</math></p> <p><math>(\quad + \quad) \times (\quad + \quad)</math></p> <table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; width: 40px; height: 40px;"></td> <td style="width: 40px; height: 40px;"></td> <td style="width: 40px; height: 40px;"></td> <td style="width: 40px; height: 40px;"></td> </tr> </table> <p style="text-align: center; border: 1px solid black; width: 60px; height: 20px; margin: 0 auto;"></p>								
20	1600	20	✓														
9	720	9	✓														

Figure C4. Example of 2 X 2 graphic organizer, grading and guided practice for self-evaluation sheet, goal setting and scaffold use.



Appendix C (Continued)

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

**Sample**

Date: 1-19-19 ✓

I completed # 3 problems out of total # 4 problems. ✓

I scored # 18 points ÷ 40 × 100 = 45 % correct. (Teacher adds)

Yesterday I did **Better** **Worse** **Same** P-2

Did I make a mistake with setting up the area model grid? **Yes** P **No**

Didn't set up the grid with the correct number of squares.

Didn't write the correct expanded number for each place value. P

Other: \_\_\_\_\_

**+** **Did I miscalculate?** **Yes** **No** ✓

I didn't carry over correctly.

I didn't add the digits correctly.

I didn't line up the numbers so they were in the correct place value.

I used mental math and I didn't keep track of the products that I was adding. ✓

Other: \_\_\_\_\_

**×** **Yes** **No** ✓

I didn't multiply single digits correctly.

I didn't multiply by factors of 10 correctly. ✓

Other: \_\_\_\_\_

**I didn't finish the assignment because...** **Yes** **No** P

I was frustrated.

I was confused by: \_\_\_\_\_ P

I was distracted. P

Other: \_\_\_\_\_

Figure C5. Daily self-reflection based on example in Figure C4.

Appendix C (Continued)

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

**Goals for Week** Date: 1-19-19 Score: 11 /20

**Monday**

✓ Do I need to **finish** my work?  Yes  No



I will:

P  Ask for a **2 minute break**, if needed.

Ask for **help** if I am confused. *+ Wednesday*

Use music or headphones to focus.

Other: \_\_\_\_\_

✓ Did I make **mistakes**?  Yes  No

I will:

Use a **reference sheet** if I'm confused.

Use a **graphic organizer**.



Use a **calculation tool**.

✓  **Check my work** for silly mistakes and miscalculations.

Be as **neat** as possible.

**Show more of my work**.

Other: \_\_\_\_\_


  


✓ If I got a **high score**, do I want to challenge myself?  Yes  No

I will:

✓  Not use a graphic organizer and **set up my own grid**.

Ask for **harder problems** (e.g. 3 by 3 digits).



The below will be graded 2 points, each in

	Yes (Y), No (N), Sort of/Stayed the Same (S)			
	Tuesday	Wednesday	Thursday	Friday
Did I use my strategy goals?	Y +2	N +2	Y +2	Y +2
Did my percent accuracy increase?	Y +2	N +2	Y +2	Y +2
Do I want to keep my goals? (If no, make notes above.)	Y +2	Y P	Y +2	N +2

level of questions above...

Figure C6. Daily goal setting sheet (for 1 week) based on example Figure C4. On Monday they



Appendix C (Continued)

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

<p>① <math>97 \times 38 = ?</math></p> <p>Didn't carry over</p> <p><math>(90 + 7) \times (30 + 8)</math></p> <p><math>\times 90 + 7</math></p> <table border="1"> <tr> <td>30</td> <td>2700</td> <td>210</td> </tr> <tr> <td>8</td> <td>720</td> <td>56</td> </tr> </table> <p>+ 2700</p> <p>+ 210</p> <p>+ 720</p> <p>+ 56</p> <p>2,686</p> <p>Final Product</p>	30	2700	210	8	720	56	<p>② <math>37 \times 86 = ?</math></p> <p>Didn't multiply correctly</p> <p><math>(30 + 7) \times (80 + 6)</math></p> <p><math>\times 80 + 6</math></p> <table border="1"> <tr> <td>30</td> <td>2400</td> <td>180</td> </tr> <tr> <td>7</td> <td>56</td> <td>36</td> </tr> </table> <p>2400</p> <p>180</p> <p>56</p> <p>36</p> <p>2672</p> <p>Final Product</p>	30	2400	180	7	56	36
30	2700	210											
8	720	56											
30	2400	180											
7	56	36											
<p>③ <math>49 \times 69 = ?</math></p> <p>Didn't add correctly</p> <p><math>(40 + 9) \times (60 + 9)</math></p> <p><math>\times 40 \quad 9</math></p> <table border="1"> <tr> <td>60</td> <td>2400</td> <td>540</td> </tr> <tr> <td>9</td> <td>360</td> <td>81</td> </tr> </table> <p>12,400</p> <p>540</p> <p>360</p> <p>81</p> <p>3,391</p> <p>Final Product</p>	60	2400	540	9	360	81	<p>④ <math>52 \times 74 = ?</math></p> <p>Didn't line up addends correctly.</p> <p><math>\times 50 \quad 2</math></p> <table border="1"> <tr> <td>70</td> <td>3500</td> <td>140</td> </tr> <tr> <td>4</td> <td>200</td> <td>8</td> </tr> </table> <p>3,500</p> <p>200</p> <p>140</p> <p>+ 8</p> <p>6,980</p> <p>Final Product</p>	70	3500	140	4	200	8
60	2400	540											
9	360	81											
70	3500	140											
4	200	8											

Figure C7. Graphic organizer (1 of 2) used for modeling and guided practice scaffold use, self-evaluation, and goal setting sheets. Notes on errors were removed for instruction.

Appendix C (Continued)

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

<p>① <math>34 \times 57 = ?</math></p> <p>Didn't multiply by 10s correctly (2)</p> $\begin{array}{r} \times 50 + 7 \\ 30 \quad 1500 \quad 210 \\ 4 \quad 20 \quad 28 \end{array}$ <p>+ + 1,700 + 210 + 28</p> <p>partial products</p> <p>Final product</p> <p>□ 1,938</p>	<p>② <math>65 \times 44 = ?</math></p> <p>Left out partial product, Mental Math. (42 + 1)</p> $\begin{array}{r} \times 40 + 4 \\ 60 \quad 2,400 \quad 240 \\ 5 \quad 200 \quad 20 \end{array}$ <p>+ 2,420 + 200</p> <p>2,620</p> <p>Answer</p>
<p>③ <math>29 \times 81 = ?</math></p> <p>Did write correct expanded number (1)</p> $\begin{array}{r} \times 80 + 1 \\ 20 \quad 160 \quad 20 \\ 9 \quad 72 \quad 9 \end{array}$ <p>169 + 92 171</p> <p>Answer</p>	<p>④ Didn't complete assignment. - ?</p> <p>( + ) X ( + )</p> $\begin{array}{r} \times \\ \square \quad \square \\ \square \quad \square \end{array}$ <p>+</p> <p>□</p>

Figure C8. Graphic organizer (2 of 2) used for modeling and guided practice scaffold use, self-evaluation, and goal setting sheets. Notes on errors will be removed for instruction.

Appendix C (Continued)

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

<p>① <math>334 \times 57 = ?</math></p> <p>X 300 + 30 + 4</p> <table border="1"> <tr> <td>50</td> <td>15,000<sup>+2</sup></td> <td>1,500<sup>+2</sup></td> <td>200</td> </tr> <tr> <td>7</td> <td>2,100<sup>+2</sup></td> <td>350<sup>+2</sup></td> <td>27<sup>28</sup></td> </tr> </table> <p>15,000 2,100 1,500 350 20 + 27</p> <hr/> <p>18,997 <sup>0/2</sup> (incorrect answer)</p> <p>6 / 10</p>	50	15,000 <sup>+2</sup>	1,500 <sup>+2</sup>	200	7	2,100 <sup>+2</sup>	350 <sup>+2</sup>	27 <sup>28</sup>	<p>② <math>672 \times 41 = ?</math></p> <p>X 600 + 70 + 2</p> <table border="1"> <tr> <td>40</td> <td>24,000<sup>+2</sup></td> <td>2,800<sup>+2</sup></td> <td>80<sup>+2</sup></td> </tr> <tr> <td>1</td> <td>600<sup>+2</sup></td> <td>70<sup>+2</sup></td> <td>2<sup>+2</sup></td> </tr> </table> <p>24,800 <sup>1/2</sup> lining up place values 672 + 80</p> <hr/> <p>243,552 <sup>0/2</sup> incorrect answer.</p> <p>7 / 10</p>	40	24,000 <sup>+2</sup>	2,800 <sup>+2</sup>	80 <sup>+2</sup>	1	600 <sup>+2</sup>	70 <sup>+2</sup>	2 <sup>+2</sup>
50	15,000 <sup>+2</sup>	1,500 <sup>+2</sup>	200														
7	2,100 <sup>+2</sup>	350 <sup>+2</sup>	27 <sup>28</sup>														
40	24,000 <sup>+2</sup>	2,800 <sup>+2</sup>	80 <sup>+2</sup>														
1	600 <sup>+2</sup>	70 <sup>+2</sup>	2 <sup>+2</sup>														

Figure C9. Example of 2 X 3 partial graphic organizer and grading.

Appendix C (Continued)

Examples for Direct Instruction, Guided Practice, Scaffolds and Scoring

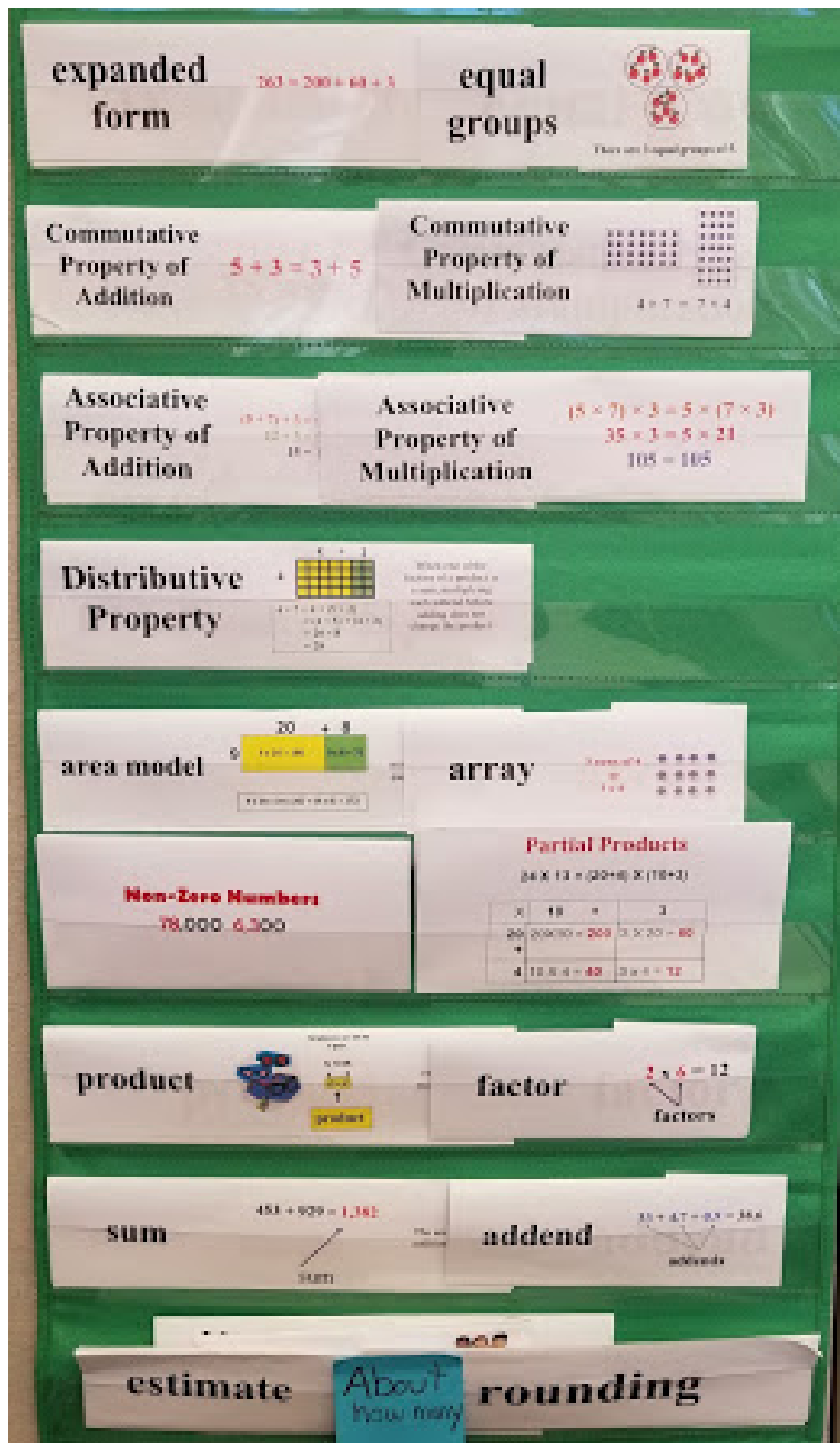


Figure C10. Word wall for use during instruction and peer modeling-self instruction.

## Silly Mistakes Checklist

### Problem Set up:

- Did I **copy** the problem correctly?
- Did I **set up** the multiplication grid correctly?
- Did I write the correct **expanded number** for each place value?

### Multiplication for Partial Products

- Did I **multiply** single digits correctly? (Should I double check with a multiplication table?)
- Did I multiply by factors of 10 correctly? (The number of zeros match the factors.)
- Did I put commas in my answers, when needed?

### Addition of Partial Products

- Did I line up the place values correctly?
- Did I add the digits correctly?
- If I did mental math, did I do it correctly? Did I leave off any numbers?



Figure C11. Silly Mistakes Checklist. This can be used by students who tend to make avoidable mistakes.

Appendix D  
Results - Figures

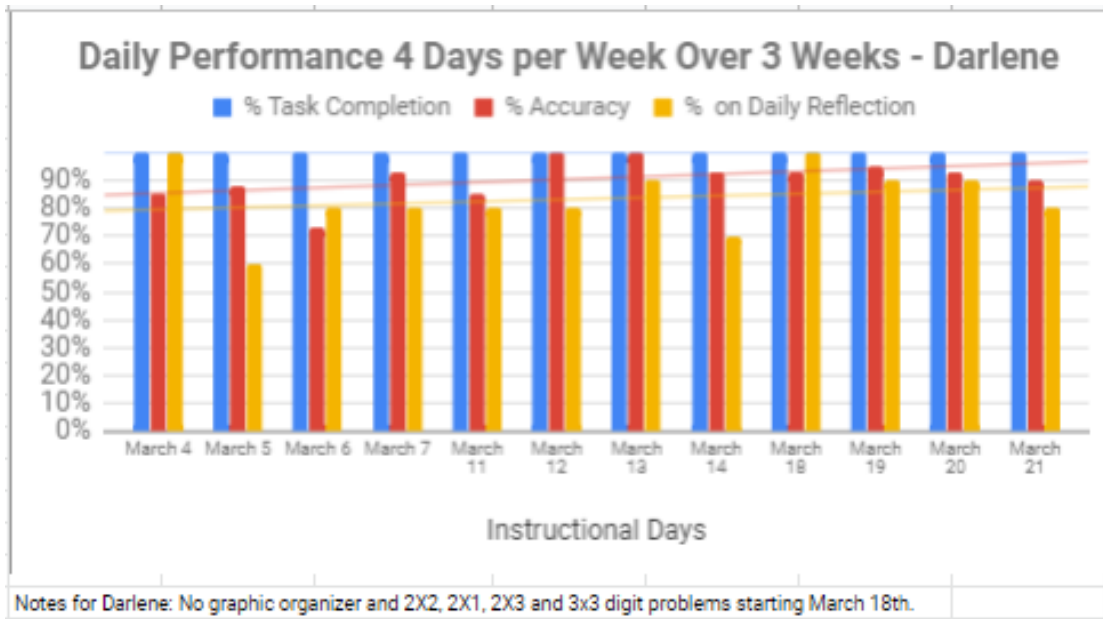


Figure D1. Daily performance: percent task completion, computation accuracy and daily reflection accuracy for Anne.

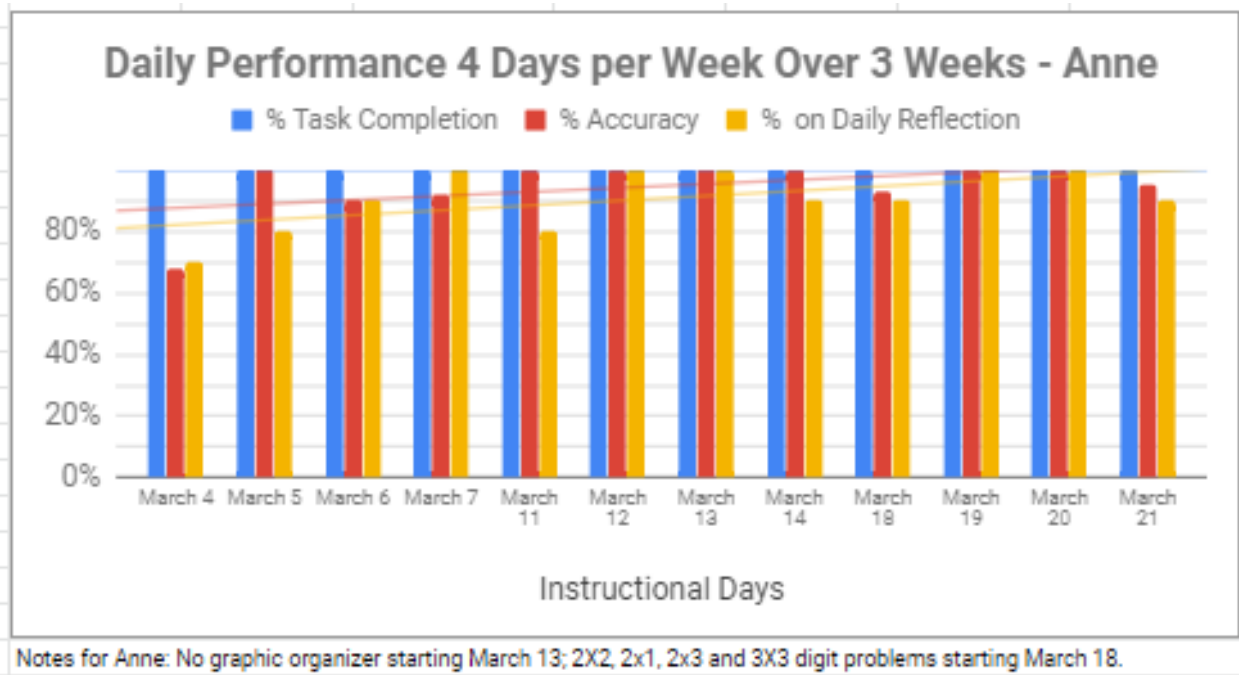


Figure D2. Daily performance: percent task completion, computation accuracy and daily reflection accuracy for Darlene.



Appendix D (Continued)

Results - Figures

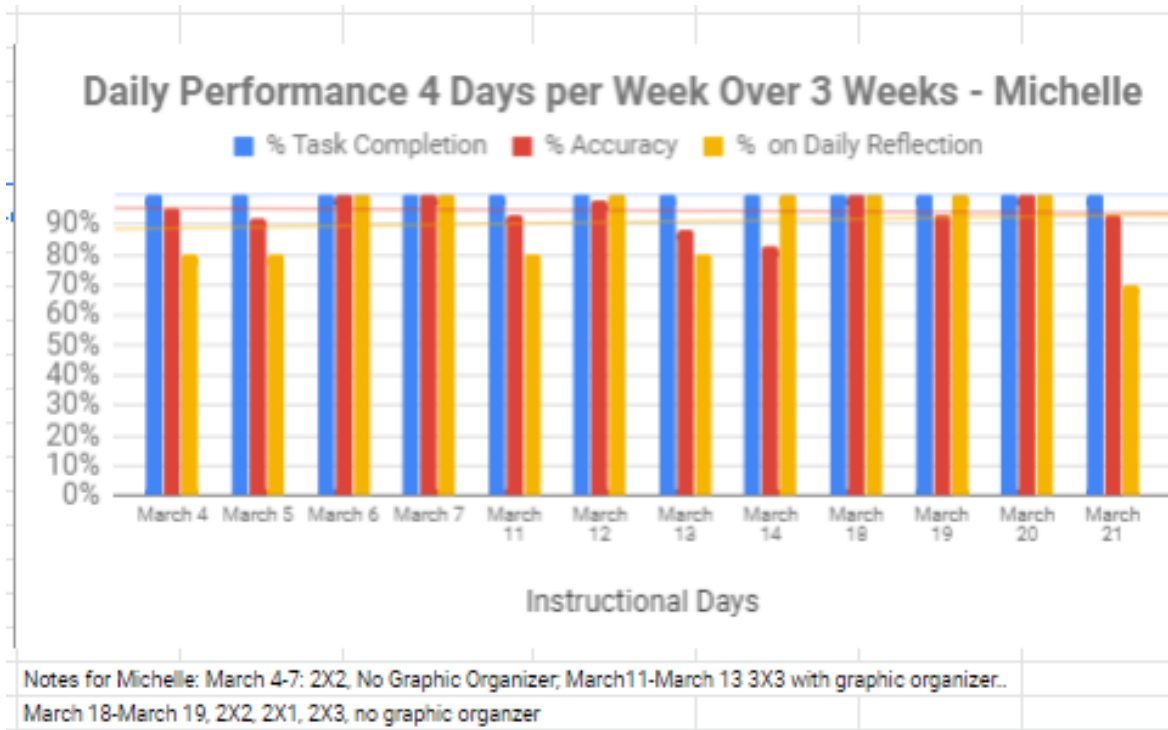


Figure D3. Daily performance: percent task completion, computation accuracy and daily reflection accuracy for Michelle.

## Appendix E

### Results

Table E1

*Average Accuracy Comparison.*

	Average Accuracy on Weekly Review Tests Fall 2018 (n=12)	Average Accuracy on New Material Multidigit Multiplication (n=12)	Difference
Michelle	85%	95%	+10
Darlene	74%	91%	+17
Anne	87%	95%	+8